PUBLIC SCHOOL DARBHANGA
SESSION (2020-21)
CLASS-VI
MATHEMATICS
POLYNOMIALS
Worksheet no.3(answer key)

1. **Determine which of the following polynomials has (x + 1) a factor:**

(i) x^3+x^2+x+1

Solution:

Let p(x)= x^3+x^2+x+1

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(-1)=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1$
$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let p(x)= $x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$
$=1-1+1-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii)$x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 +$
$x + 1$ The zero of x+1 is -1.

p(−1)=(−1)4+3(−1)3+3(−1)2+(−1)+1
$=1-3+3-1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv)$x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let p(x)= $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$
$=-1-1+2+\sqrt{2}+\sqrt{2}$
$= 2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2.**Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:**

(i) $p(x)=2x^3+x^2-2x-1$, g(x) = x + 1

Solution:

$p(x)= 2x^3+x^2-2x-1$,
g(x) = x + 1 g(x)=0

$\Rightarrow x+1=0$
$\Rightarrow x=-1$

∴Zero of g(x) is -1.

Now,
$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$
$=-2+1+2-1$
$=0$

∴By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, g(x) = x + 2

Solution:

p(x)=x3+3x2+3x+1, g(x) = x + 2
g(x)=0

$\Rightarrow x+2=0$
$\Rightarrow x=-2$

∴Zero of g(x) is -2.

Now,
$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$
$=-8+12-6+1$
$=-1\neq 0$

∴By factor theorem, g(x) is not a factor of p(x).

(iii)$p(x)=x^3-4x^2+x+6$, g(x) = x – 3

Solution:

$p(x)= x^3-4x^2+x+6$, g(x) = x -3
g(x)=0

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3. Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

3.Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow(1)^2+(1)+k=0$

$\Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0$

$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0$

$\Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$

$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2-3(1)+k=0$

$\Rightarrow k-3+k=0$

$\Rightarrow 2k-3=0$

$K=\frac{3}{2}$

4. Factorize:

(i) $12x^2 – 7x+1$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-7 and product=1×12=12
We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4=12]

$12x^2–7x+1=12x^2-4x-3x+1$
$=4x (3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=7 and product=2× 3=6
We get 6 and 1 as the numbers[6+1=7and6×1=6]
$2x^2+7x+3 =2x^2+6x+1x+3$
$=2x (x+3)+1(x+3)$
$= (2x+1)(x+3)$

(iii)$6x^2+5x-6$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=5 and product=6× −6= -36
We get -4 and 9 as the numbers [-4+9=5 and -4× 9=-36] $6x^2+5x-6=6x^2+ 9x – 4x – 6$
$=3x (2x + 3) – 2 (2x + 3)$

$= (2x + 3) (3x – 2)$

(iv) $\mathbf{3x^2 - x - 4}$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=$3\times -4 = -12$

We get -4 and 3 as the numbers [-4+3=-1 and -4× 3=-12]

$3x^2 - x - 4 = 3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x-4) + 1(3x-4)$

$= (3x-4)(x+1)$

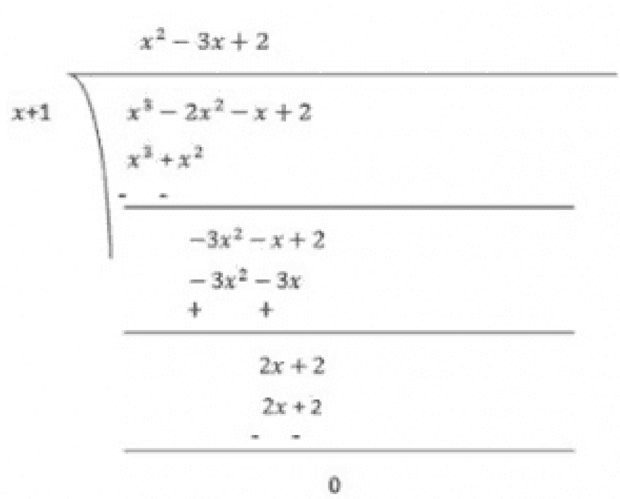
5. Factorize:

(i) $\mathbf{x^3 - 2x^2 - x + 2}$

Solution:

Let $p(x) = x^3 - 2x^2 - x + 2$ Factors of 2 are ±1 and ± 2 By trial method, we find that p(1) = 0

So, (x+1) is factor of p(x)



Now,

$p(x) = x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 1 + 1 + 2$

$= 0$

Therefore, (x+1) is the factor of p(x)

Now, Dividend = Divisor × Quotient +

Remainder $(x+1)(x^2–3x+2)$

$=(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x-2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5 By trial method, we find that p(5) = 0

So, (x-5) is factor of p(x)

Now,

$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, (x-5) is the factor of p(x)

Quotient: $x^2 + 2x + 1$

Divisor: x-5

$x^3 - 3x^2 - 9x - 5$

$x^3 - 5x^2$

$-\quad +$

$2x^2 - 9x - 5$

$2x^2 - 10x$

$-\quad +$

$x - 5$

$x - 5$

$-\quad +$

0

Now, Dividend = Divisor × Quotient + Remainder

$$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$$
$$= (x-5)(x(x+1)+1(x+1))$$
$$= (x-5)(x+1)(x+1)$$

(iii)$x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

$$p(x) = x^3+13x^2+32x+20$$
$$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$$
$$= -1+13-32+20$$
$$= 0$$

Therefore, $(x+1)$ is the factor of $p(x)$

Quotient: $x^2 + 12x + 20$

Divisor: $x+1$

$x^3 + 13x^2 + 32x + 20$

$x^3 + x^2$

$-\quad -$

$12x^2 + 32x + 20$

$12x^2 + 12x$

$-\quad -$

$20x + 20$

$20x + 20$

$-\quad -$

0

Now, Dividend = Divisor × Quotient +

Remainder $(x+1)(x^2+12x+20)$

$=(x+1)(x^2+2x+10x+20)$

$=(x+1)x(x+2)+10(x+2)$

$=(x+1)(x+2)(x+10)$

(iv) $\mathbf{2y^3+y^2-2y-1}$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors = $2\times(-1)$= -2 are ±1 and ±2

By trial method, we find that

$p(1) = 0$

So, (y-1) is factor of p(y)

Now,

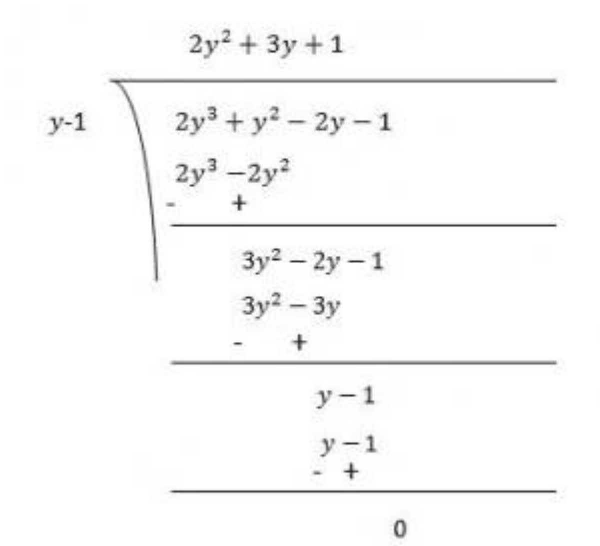
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$=2+1-2$

$=0$

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
(y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
&= (y-1)(2y(y+1)+1(y+1)) \\
&= (y-1)(2y+1)(y+1)
\end{aligned}
$$