Some useful question on Multiplication

Q.1. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ Then show that

$$A^2 - 4A + 7I = 0$$

Using this result calculate A^5.

Sol^n :-

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$LHS = A^2 - 4A + 7I$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} = 0 = RHS$$

$$\therefore A^5 = A^2 \cdot A^2 \cdot A$$

Q. 2 If A is a square matrix such that $A^2 = I$, then find the value of

$$(A-I)^3 + (A+I)^3 - 7A.$$

Sol^n : → $(A-I)^3 + (A+I)^3 - 7A$

$$= A^3 - I^3 - 3A^2 I + 3AI^2 + A^3 + I^3 + 3A^2 I + 3AI^2 - 7A$$

$$= 2A^3 + 6AI^2 - 7A$$

$$= 2(A^2)A + 6A - 7A$$

$$= 2IA - A$$

$$= 2A - A = A$$

Q. 3 Find AB if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

Sol^n

$$AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times3 - 1\times0 & 0\times5 - 1\times0 \\ 0\times3 + 2\times0 & 0\times5 + 2\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-0 & 0-0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Transpose of a Matrix

Let $A = [a_{ij}]_{m \times n}$ be a matrix, then we define the transpose of A as A^T or A' $= [a_{ji}]_{n \times m}$.

Properties of transpose of A

(i) $(A^T)^T = A$

(ii) $(KA)^T = KA^T$

(iii) $(A \pm B)^T = A^T \pm B^T$

(iv) $(AB)^T = B^T A^T$

Q. 1 $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then find the value of α if $A^T + A = I$.

Soln:- If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$\because$ $A^T + A = I$.

$$\Rightarrow \begin{bmatrix} \cos\alpha & +\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & -\sin\alpha \\ +\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad 2\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{2} = \cos \pi/3$$

$$\alpha = 2n\pi \pm \pi/3 \quad \text{Ans}$$

Q. If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$

then find the value of a, b, c s.t. $A^T.A = I$

Sol^n

$$\because \quad A^T . A = I$$

$$\begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} \begin{bmatrix} a & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a^2 & 0 & 0 \\ 0 & 6b^2 & 0 \\ 0 & 0 & 3c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad 2a^2 = 1, \quad 6b^2 = 1, \quad 3c^2 = 1$$

$$a = \pm \frac{1}{\sqrt{2}}, \quad b = \pm \frac{1}{\sqrt{6}}, \quad c = \pm$$