Potential due to an electric dipole

- We already know that electric dipole is an arrangement which consists of two equal and opposite charges +q and -q separated by a small distance 2a.
- Electric dipole moment is represented by a vector **p** of magnitude 2qa and this vector points in direction from -q to +q.

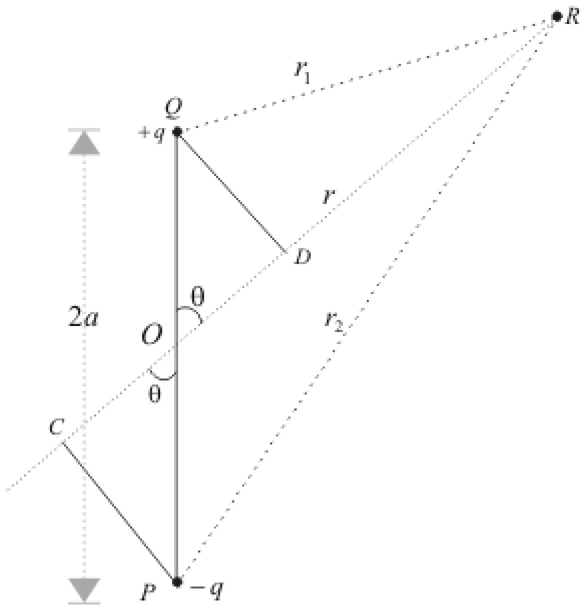

Figure 5

- Since electric potential obeys superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges +q and -q. Thus

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) \qquad (15)$$

where r_1 and r_2 respectively are distance of charge +q and -q from point R.

- Now draw line PC perpendicular to RO and line QD perpendicular to RO as shown in figure. From triangle POC
cosθ=OC/OP = OC/a
therefore OC=acosθ similarly OD=acosθ
Now ,
r_1 = QR≅RD = OR-OD = r-acosθ
r_2 = PR≅RC = OR+OC = r+acosθ

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right)$$

since magnitude of dipole is
|**p**| = 2qa

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2 - a^2\cos^2\theta}\right) \qquad (16)$$

- If we consider the case where r>>a then

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad (17)$$

again since pcosθ= **p·r^** where, **r^** is the unit vector along the vector OR then electric potential of dipole is

$$V = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} \qquad (18)$$

Statement of Gauss's Law

"ELectric flux through any surface enclosing charge is equal to q/ε_0 , where q is the net charge enclosed by the surface"

mathematically,

$$\oint \mathbf{E}\cdot d\mathbf{a} = \frac{q}{\varepsilon_0} \qquad (11)$$

where q_{enc} is the net charge enclosed by the surface and **E** is the total electric field at each point on the surface under consideration.

Coulomb's law can be derived from Gauss's law.

- Consider electric field of a single isolated positive charge of magnitude q as shown below in the figure.

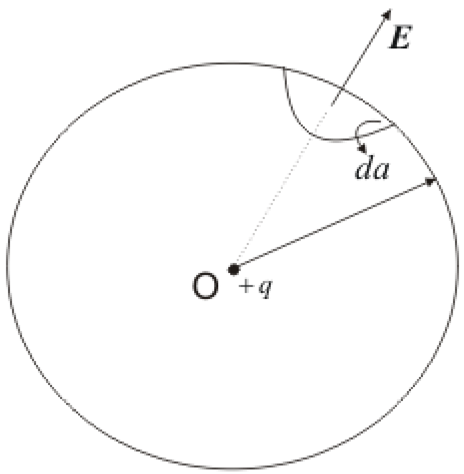

Figure 4

- Field of a positive charge is in radially outward direction everywhere and magnitude of electric field intensity is same for all points at a distance r from the charge.
- We can assume Gaussian surface to be a sphere of radius r enclosing the charge q.
- From Gauss's law

$$\oint \mathbf{E}\cdot d\mathbf{a} = E\oint da = \frac{q_{enc}}{\varepsilon_0}$$

since **E** is constant at all points on the surface therefore,

$$EA = \frac{q}{\varepsilon_0}$$

or,

$$E = \frac{q}{\varepsilon_0 A}$$

surface area of the sphere is $A=4\pi r^2$

thus,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

- Now force acting on point charge q' at distance r from point charge q is

$$F = q'E$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}$$

This is nothing but the mathematical statement of Coulomb's law.

Electric field due to line charge

- We can assume Gaussian surface to be a right circular cylinder of radius r and length l with its ends perpandicular to the wire as shown below in the figure.

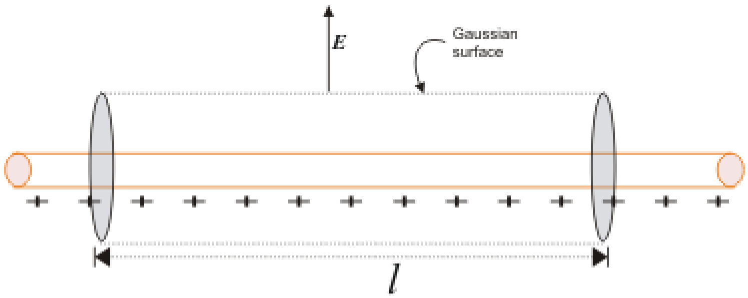

Figure 5. Cylindrical Gaussian surface for calculation of electric field due to line charge

- λ is the charge per unit length on the wire. Direction of E is perpendicular to the wire and components of E normal to end faces of cylinder makes no contribution to electric flux. Thus from Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$$

- Now consider left hand side of Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = E\oint da$$

Since at all points on the curved surface E is constant. Surface area of cylinder of radius r and length l is $A=2\pi rl$ therefore,

$$\oint \mathbf{E} \cdot d\mathbf{a} = E(2\pi rl)$$

- Charge enclosed in cylinder is q=linear charge density x length l of cylinder,

or, $q=\lambda l$

From Gauss's law

$$\oint \boldsymbol{E} \cdot \boldsymbol{da} = \frac{q}{\varepsilon_0}$$

$$or, E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$$\Rightarrow E \propto \frac{\lambda}{r}$$

Electric field due to charged solid sphere

- Gaussian surface would be a sphere of radius r>R concentric with the charged solid sphere shown below in the figure. From Gauss's law

  $$\oint \boldsymbol{E} \cdot \boldsymbol{da} = \frac{q_{enc}}{\varepsilon_0}$$

  where q is the charge enclosed.
- Charge is distributed uniformly over the surface of the sphere. Symmetry allows us to extract **E** out of the integral sign as magnitude of electric field intensity is same for all points at distance r>R.
- Since electric field points radially outwards we have

  $$\oint \boldsymbol{E} \cdot \boldsymbol{da} = E\oint da$$

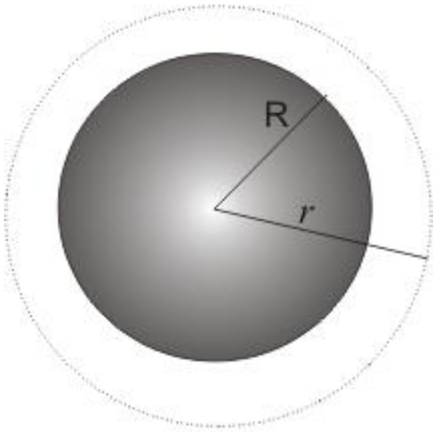

Figure 6

also as discussed magnitude of **E** is constant over Gaussian surface so,

$$E\oint da = E(4\pi r^2)$$

where $4\pi r^2$ is the surface area of the sphere.
Again from Gauss's law we have

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Thus we see that magnitude of field outside the sphere is exactly the same as it would have been as if all the charge were concentrated at its centre.

Electric field due to an infinite plane sheet of charge

- Consider a thin infinite plane sheet of charge having surface charge density σ(charge per unit area).

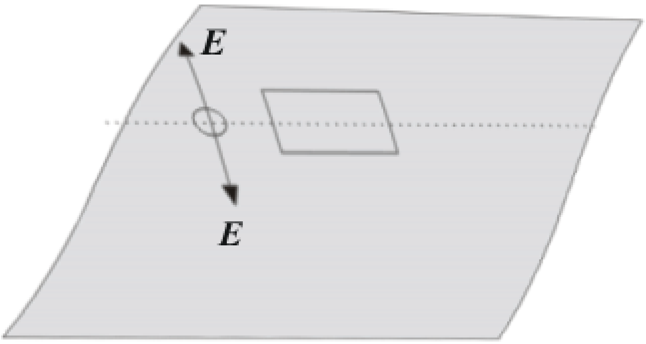

Figure 7

By symmetry we find that Eon either side of sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.

No field lines crosses the side walls of the Gaussian pillbox i.e., component of E normal to walls of pillbox is zero.

We now apply Gauss's law to this surface

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$$

in this case charge enclosed is
$q=\sigma A$
where A is the area of end face of Gaussian pillbox.

E points in the direction away from the plane i.e., E points upwards for points above the plane and downwards for points below the plane. Thus for top and bottom surfaces,

$$\oint \mathbf{E} \cdot d\mathbf{a} = 2A|\mathbf{E}|$$

thus
$2A|E|=\sigma A/\varepsilon_0$
or,
$|E|=\sigma/2\varepsilon_0$
Here one important thing to note is that magnitude of electric field at any point is independent of the sheet and does not decrease inversely with the square of the distance. Thus electric field due to an infinite plane sheet of charge does not falls of at all.