Electrostatics

- Electrostatics is the study of charges at rest.
- The intrinsic property of fundamental particle of matter which give rise to electric force between objects is called charge.
- Charging a body can be done by friction, induction and conduction.

Basic Properties of charges:

- Like charges repel and unlike charges attract.
- Charges are additive in nature i.e., $Q=\sum_{i=1}^{n} q_i$
- Charges are quantized. i.e., Q= ± ne [n=1,2,3,… & e=1.602 X10^{-19} C]
- Charge on a body is independent of velocity of the body.
- Charge is conserved.

- The sensitive device which is used to identify whether the body is charged or not is called electroscope.

- The charge is said to be one coulomb when it is separated from similar charge by one meter experiences a force of repulsion 9×10^9 N.
- The period of revolution of charge q_1 of mass m about charge q_2 along the circular path of radius r is $T = \sqrt{\frac{16\pi^3\epsilon_0 m r^3}{q_1 q_2}}$
- Principle of superposition: $\vec{F}_{total} = \sum_{i=1}^{n} \vec{F_i}$[Vector sum of individual forces]

$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{1}{4\pi\varepsilon_o}\frac{q_1q_3}{r_{13}^2}\hat{r}_{13} + \ldots$$

- Uniform Charge distribution:

Linear charge distribution: $\lambda = \frac{\Delta q}{\Delta l}$ [$\lambda \Rightarrow$ linear charge density Unit Cm^{-1}]

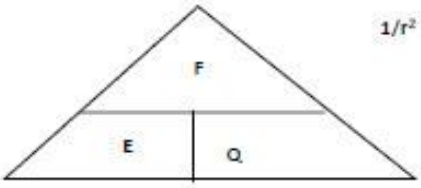
Surface charge distribution: $\sigma = \frac{\Delta q}{\Delta S}$ [$\sigma \Rightarrow$ surface charge density Unit Cm^{-2}]

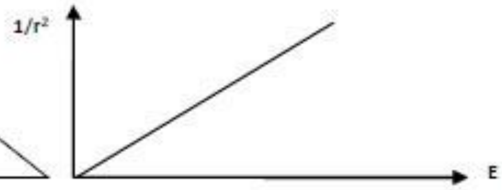
Volume charge distribution: $\rho = \frac{\Delta q}{\Delta V}$ [$\rho \Rightarrow$ Volume charge density Unit Cm^{-3}]

- Force due to continuous charge distribution:

$$\vec{F} = \frac{q_o}{4\pi\epsilon_0}\left[\int_L \frac{\lambda\, dl}{r^2} + \int_S \frac{\sigma\, dS}{r^2} + \int_V \frac{\rho\, dV}{r^2}\right]\hat{r}$$

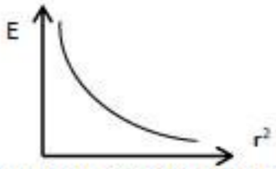
- The comparison of electrostatic and gravitational forces between electron and proton is $\frac{F_e}{F_g} = \frac{ke^2}{Gm_p m_e} = 2.27 \times 10^{39}$.





Note: *In the above triangle the quantity shown at the vertex, could be arrived by multiplying the quantities shown at the base, i.e. F=E X Q.*
Any one of the quantity shown at the base is given by the ratio of the quantities shown at vertex & the other quantity shown at the base, i.e. E=F/Q or Q= F/E.

- Electric field: Force experienced by a unit positive charge. It is a vector. SI unit is NC^{-1}. $\vec{E} = \lim_{q_o \to 0} \frac{\vec{F}}{q_o}$



- Field due to a point charge Q at r is $\vec{E} = \frac{kQ}{r^2}\hat{r}$
- Principle of superposition: $\vec{E}_{total} = \sum_{i=1}^{n} \vec{E_i}$[vector sum of individual fields]
- Electric field due to continuous charge distribution:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\int_L \frac{\lambda\, dl}{r^2} + \int_S \frac{\sigma\, dS}{r^2} + \int_V \frac{\rho\, dV}{r^2}\right]\hat{r}$$

- **Dipole :** Two equal and opposite charges separated by a small distance.
- Dipole moment: Product of magnitude of either charge and distance of separation between them. It is a vector. SI unit: Cm, $\vec{p} = (Q)\, 2\vec{a}$; direction of $\vec{p}$ is from negative charge to positive charge along the straight line joining both the charges.
- Dipole in a uniform electric field experiences no net translating force but experiences a

torque. $\vec{\tau}=\vec{p}\times\vec{E} \Rightarrow \vec{\tau}=|\vec{p}||\vec{E}|\sin\theta\,\hat{n}$

- If $\theta = 0°$ ⇒ stable equilibrium; If $\theta = 180°$ ⇒ unstable equilibrium.
- Electric field due to a short dipole

♦ **Properties of electric field lines:**

✓Arbitrarily starts from +ve charge and end at –ve charge

✓ Continuous, without any breaks, never form closed loops

✓Never intersect

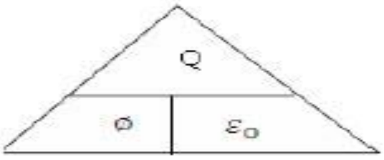
✓Relative closeness of the field lines represents the magnitude of the fieldstrength.

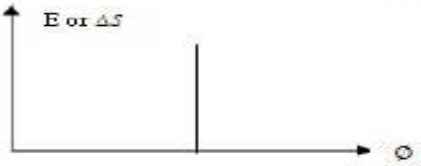
✓For a set of two like charges – lateral pressure in between

✓For a set of two unlike charges – longitudinal contraction in between.

♦ **Area vector: The vector quantity representing the area of a surface whosemagnitude is equal to the magnitude of the area and direction is perpendicular to the surface.**

- Electric flux: $\varnothing=\overrightarrow{\Delta S}.\vec{E}=|\vec{E}||\overrightarrow{\Delta S}|cos\theta$; It is a scalar; SI unit: N m^2C^{-1} or Vm.
- Gauss' theorem in electrostatics: $\varnothing_{total}=\oint_s \vec{E}.\overrightarrow{dS}=\frac{q_{total}}{\epsilon_0}$





ϕ is independent of $\vec{E}$ & $\vec{\Delta s}$

- Applications of Gauss' theorem for uniform charge distribution:

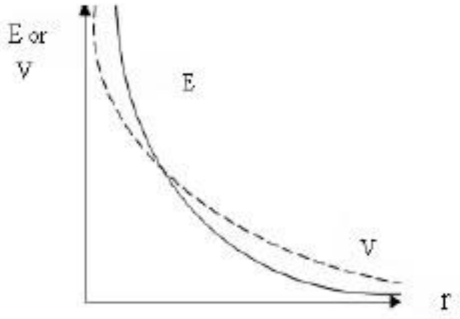
Expression for	**Infinite Linear**	**Infinite plane sheet**	**Thin spherical shell**
Flux Ø	$\frac{\lambda l}{\epsilon_0}$	$\frac{\sigma s}{\epsilon_0}$	$\frac{\sigma 4\pi r^2}{\epsilon_0}$
Magnitude of Field E	$\frac{\lambda}{2\pi r\epsilon_0}$	$\frac{\sigma}{\epsilon_0}$	$\frac{Q}{4\pi r^2\epsilon_0}$[for points on/outside the shell] ZERO [for points inside the shell]
Charge density	$\lambda=\frac{\Delta q}{\Delta l}$	$\sigma=\frac{\Delta q}{\Delta S}$	$\frac{\sigma}{4\pi r^2}$

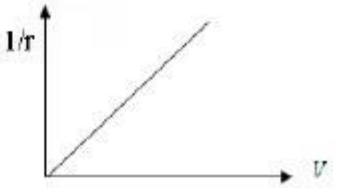
- Electrostatic Potential: Work done per unit positive Test charge to move it from infinity to that point in an electric field. It is a scalar. SI unit: J/C or V

$$V = W / q_o$$

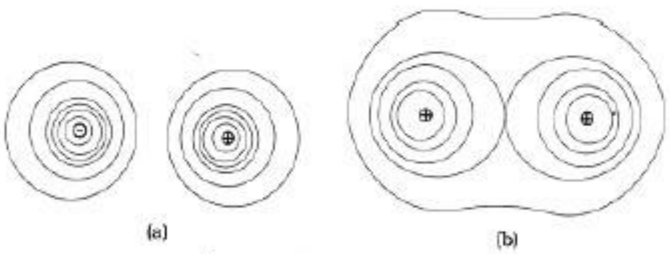
Electric potential for a point charge: $V = \frac{kq}{r}$

- The electrostatic potential at any point in an electric field is said to be one volt when one joule of work is done in bringing one unit charge from infinity to that point.
- The electric field intensity at any point is the negative gradient of potential at that point. $E = -dV/dr$. $V(\vec{r}) = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$
- As E= $-\frac{dV}{dr}$ If Vis constant, E$\propto \frac{1}{r}$and if E is constant, V$\propto r$





- Electric field is conservative. This means that the work done is independent of the path followed and the total work done in a closed path is zero.
- Potential due to a system of charges: $V_{total} = \sum_{i=1}^{n} \frac{kq_i}{r_i}$
- Potential due to a dipole at a point
  - on its axial line: $V_{axial} = \frac{k|\vec{p}|}{r^2}$ [or]$\frac{k|\vec{p}|}{r^2} cos\theta$
  - on its equatorial line: $V_{eq} = 0$
- Potential difference $V_A - V_B = kq\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$
- Potential energy of two charges: $U = \frac{kq_1q_2}{r}$
- Potential energy of a dipole : $U = -\vec{p}.\vec{E} = p\,E\ [cos\theta_1\ -cos\theta_2]$
- Equipotential surfaces: The surfaces on which the potential is same everywhere.
  - ✓ Work done in moving a charge over an equipotential surface is zero.
  - ✓ No two equipotential surfaces intersect.
  - ✓ Electric field lines are always perpendicular to the equipotential surfaces.
  - ✓ The relative density of equipotential surface gives intensity of electric field in that region.



Some equipotential surfaces for (a) a dipole, (b) two identical positive charges.

- **Electrostatics of conductors**

(i) Inside a conductor Electrostatic field is zero

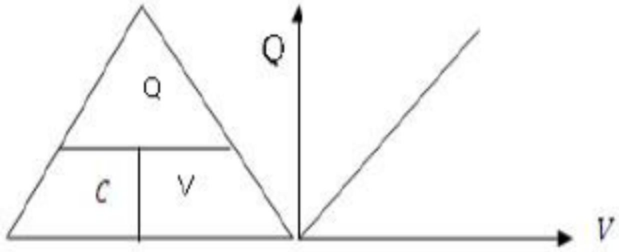
(ii) On the surface E is always Normal to the surface

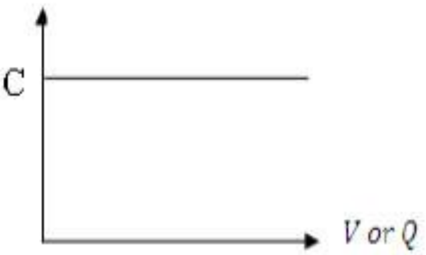
(iii) No excess charge resides inside the conductor

(iv) Charge distribution on the surface is uniform if the surface is smooth

(v) Electric field is zero in the cavity of hollow conductor and potential remains constant which is equal to that on the surface.

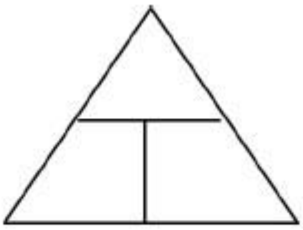
- Capacitor: An arrangement of two conductors separated by a small distance without any electrical contact between them is called capacitor.





- Capacitance: $C = \frac{Q}{V}$, Ratio of charge and potential difference. Scalar. SI unit: farad [F]. The capacitance is said to be one farad when one coulomb of charge increases the potential difference between the plates by one volt.
- Capacitance of a parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$

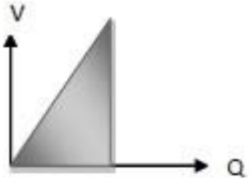
- Capacitance of a parallel plate capacitor with a dielectric medium in between:
  - $C_m = \frac{\epsilon_0 A}{\left(d-t+\frac{t}{k}\right)}$
  - If t=0 $\Rightarrow C_0 = \frac{\epsilon_0 A}{(d)}$
  - If t=d $\Rightarrow C_0 = k\frac{\epsilon_0 A}{(d)} \Rightarrow C_m = k\, C_0$



- Combination of capacitors:

  Capacitors in series: $\frac{1}{c} = \sum_{i=1}^{n} \frac{1}{c_i}$ Capacitors in parallel: $c = \sum_{i=1}^{n} c_i$

- Energy stored in capacitors: $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$
- Area shaded in the graph = U $= \frac{1}{2}QV$
- Energy density : $U_d = \frac{1}{2}\epsilon_0 E^2 = \frac{\sigma^2}{2\epsilon_0}$



- The total energy in series and parallel combinations of capacitors is additive.
- When two charged conductors are touched mutually and then separated the redistribution of charges on them is in the ratio of their capacitances.
- Introducing dielectric slab between the plates of the charged capacitor with:

Property	**Battery connected**	**Battery disconnected**
Charge	K Q_0	Q_0
Potential difference	V_0	V_0/K
Electric field	E_0	E_0/K
Capacitance	KC_0	KC_0
Energy	K times $\frac{1}{2}\epsilon_0 E^2$[Energy is supplied By battery]	1/K times $\frac{1}{2}\epsilon_0 E^2$[Energy used for Polarization]

- On connecting two charged capacitors:
  - Common Potential: $V = \frac{C_1V_1 + C_2V_2}{V_1 + V_2}$
  - Loss of energy: $\Delta U = \frac{1}{2}\frac{C_1 \times C_2}{C_1 + C_2}(V_1 - V_2)^2$

- The dielectric is the substance which is essentially an insulator but behaves like a conductor in electrostatic situation.
- The dielectric having atom or molecules whose negative charge centre is not coinciding with positive charge centre is called polar dielectric. They have permanent dipole moments in the order of 10^{-30} Cm.
- The dielectric having atom or molecules whose negative charge centre is coinciding with positive charge centre is called non-polar dielectric.
- The dipole moment developed in non-polar dielectric due to external electric field is called induced dipole moment.
- The induced dipole moment per unit volume is called Polarisation Vector. The direction of polarisation vector

is same as that of external electric filed.

- The ratio of electrostatic force in free space to that in medium OR the ratio of electrostatic field in free space to that in medium OR the ratio of absolute permittivity of medium to that of free space is called relative permittivity or dielecric constant of the medium. $\epsilon_o OR\ \kappa$.
- The ratio of polarisation to ϵ_o times the electric field intensity is called electric susceptibility. $\chi = \frac{P}{\epsilon_0 E}$. he dielectrics with constant χ are called linear dielectrics.
- The maximum external electric field the dielectric can withstand without dielectric beakdown is called dielectric strength. SI unit Vm^{-1}
- The capacitance of a spherical conductor of radius R is $C = 4\pi\epsilon_o R$.