2

POLYNOMIALS

SUMMARY

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials, respectively.
2. A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a,b,c are real numbers with $a \neq 0$.
3. If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$, is called the value of $p(x)$ at $x = k$, and is denoted by $p(k)$.
4. A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.
5. The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points, where the graph of $y = p(x)$ intersects the x-axis.
6. A polynomial $p(x)$ of degree n has at most n zeroes.
7. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
8. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

   $$\alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$$
9. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

   $$\alpha + \beta + \gamma = \frac{-b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a},$$

   and $\alpha\beta\gamma = \frac{-d}{a}$.
10. If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be

    x^2 – (sum of zeroes)x + product of zeroes

    i.e., $x^2 - (\alpha + \beta)x + \alpha\beta$.
11. If α,β,γ are the zeroes of a cubic polynomial, then cubic polynomial will be

    x^3 – (sum of zeroes)x^2 + (sum of product of zeroes taken two at a time)x – product of zeroes

    i.e., $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$.
12. The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that

    $p(x) = g(x)\, q(x) + r(x)$

    where $r(x) = 0$ or degree $r(x) <$ degree $g(x)$.

VERY SHORT ANSWERS (VSA) TYPE QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. Which of the following expressions is a polynomial?

(*a*) $x^3+\frac{1}{x^2}+\frac{1}{x}+1$ (*b*) $x^2+\sqrt{x}+1$

(*c*) $y^{-\frac{1}{2}}-3y+2$ (*d*) $\sqrt{2}y^3+\sqrt{3}y$

Sol. $x^3+\frac{1}{x^2}+\frac{1}{x}+1$ can be written as $x^3 + x^{-2} + x^{-1} + 1$.

It is not a polynomial because it contains terms having negative integral exponents.

It is not a polynomial because in $x^2+\sqrt{x}+1$**,** the power of $\sqrt{x}$ or $x^{\frac{1}{2}}$ is $\frac{1}{2}$, which is not a whole number.

It is not a polynomial because power of $y^{-\frac{1}{2}}$ is $-\frac{1}{2}$, which is not a whole number.

Yes, it is a polynomial because it satisfies the condition of a polynomial.

Hence, option (*d*) is correct.

2. If $p(x) = g(x) \times q(x) + r(x)$, then degree of $q(x)$ is always less than

(*a*) the degree of $g(x)$

(*b*) the degree of $p(x)$

(*c*) the degree of $r(x)$

(*d*) or equal to degree of $p(x)$

Sol. The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$

Where $r(x) = 0$

or degree of $r(x) <$ degree of $g(x)$.

Hence, option (*d*) is correct.

3. Polynomial of degree *n* has

(*a*) only 1 zero

(*b*) atmost *n* zeroes

(*c*) exactly *n* zeroes

(*d*) more than *n* zeroes

Sol. The polynomial $p(x)$ of degree n has atmost n zeroes.

Hence, option (*b*) is correct.

4. Graph of a quadratic polynomial is a

(*a*) straight line **(*b*) circle**

(*c*) parabola **(*d*) ellipse**

Sol. Graph of a quadratic polynomial is a parabola.

Hence, option (*c*) is correct.

5. Quadratic polynomial having zeroes 1 and –2 is

(*a*) $x^2 - x + 2$ **(*b*) $x^2 - x - 2$**

(*c*) $x^2 + x - 2$ **(*d*) $x^2 + x + 2$**

Sol. If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be

x^2 – (sum of zeroes)x + product of zeroes

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow x^2 - [1 + (-2)]x + 1(-2)$

$\Rightarrow x^2 + x - 2$

Hence, option (*c*) is correct.

6. The quadratic polynomial, the sum of whose zeroes is 0 and one zero is 5 is,

(*a*) $x^2 - 5$ **(*b*) $x^2 - 25$**

(*c*) $x^2 + 25$ **(*d*) none**

Sol. Let α and β are the zeroes of the polynomial.

It is given that, sum of zeroes = 0

i.e., $\alpha + \beta = 0$

If $\alpha = 5$, then $5 + \beta = 0 \quad \Rightarrow \quad \beta = -5$

Now, $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 0x + (-5)(5)$

$= x^2 - 25$

Hence, option (*b*) is correct.

7. The zeroes of the quadratic polynomial $x^2 + 88x + 125$ are

(*a*) both positive

(*b*) both negative

(*c*) one positive and one negative

(*d*) both equal

Sol. Let α and β be the zeroes of the given polynomial.

Then $\alpha + \beta = 88$ and $\alpha\beta = 125$

If α and β are the zeroes of a quadratic polynomial, then quadratic polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$.

This is possible only when α and β are both negative.

Hence, option (*b*) is correct.

8. If α and β are the zeroes of $2x^2 + 5x - 9$, then the value of $\alpha\beta$ is

(*a*) $\frac{-5}{2}$ **(*b*)** $\frac{5}{2}$

(*c*) $\frac{-9}{2}$ **(*d*)** $\frac{9}{2}$

Sol. We know that if α and β are the zeroes of the polynomial $ax^2 + bx + c$

then, $\alpha\beta = \frac{c}{a}$

Here $\alpha\beta = \frac{-9}{2}$

Hence, option (*c*) is correct.

9. Which of the following is a true statement?

(*a*) $x^2 + 5x - 3$ is a linear polynomial.

(*b*) $x^2 + 4x - 1$ is a binomial.

(*c*) $x + 1$ is monomial.

(*d*) $5x^2$ is a monomial.

Sol. $x^2 + 5x - 3$ is a quadratic polynomial, $x^2 + 4x - 1$ is a trinomial and $x + 1$ is a binomial. $5x^2$ is a monomial.

Hence, option (*d*) is correct.

10. If one zero of $3x^2 + 8x + k$ be the reciprocal of the other, then k is

(*a*) 3 **(*b*) –3**

(*c*) $\frac{1}{3}$ **(*d*) $\frac{-1}{3}$**

Sol. If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha\beta = \frac{c}{a}$.

Here, $a = 3$, $b = 8$, $c = k$

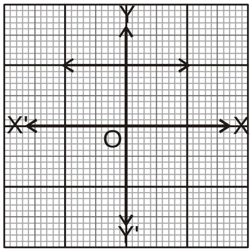
Let one zero be α and other be $\frac{1}{\alpha}$.

Then $\alpha \times \frac{1}{\alpha} = \frac{k}{3}$

$\Rightarrow\ 1 = \frac{k}{3} \qquad \Rightarrow k = 3$

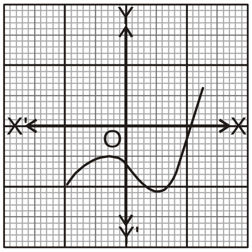
Hence, option (*a*) is correct.

11. The graph of $y = p(x)$ is given in the following figure. Find the number of zeroes of $p(x)$. [NCERT]



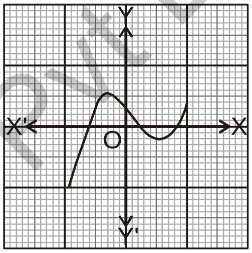
Sol. The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

12. The graph of $y = p(x)$ is given in the following figure. Find the number of zeroes of $p(x)$. [NCERT]



Sol. The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

13. The graph of $y = p(x)$ is given in the following figure. Find the number of zeroes of $p(x)$. [NCERT]



Sol. The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

14. Find a quadratic polynomial with $\sqrt{2}$ and $\frac{1}{3}$ as the sum and product of its zeroes respectively. [NCERT]

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}, \qquad ab = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$.

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}\,x + 1$.

15. Find a quadratic polynomial with 1 and 1 as the sum and product of its zeroes respectively. [NCERT]

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a},\ ab = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -1$, $c = 1$.

Therefore, the quadratic polynomial is $x^2 - x + 1$.

16. Find a quadratic polynomial with $-\frac{1}{4}$ and $\frac{1}{4}$ as the sum and product of its zeroes respectively. [NCERT]

Sol. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta = \frac{-1}{4} = \frac{-b}{a}, \; ab = \frac{1}{4} = \frac{c}{a}$$

If $a = 4$, then $b = 1$, $c = 1$.

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

17. If α and β are the zeroes of a polynomial such that α + β = – 6 and αβ = 5, then find the polynomial.

Sol. Sum of the zeroes = $\alpha + \beta = -6$

Product of the zeroes = $\alpha\beta = 5$

Required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - (-6)x + 5 = 0$

$\Rightarrow x^2 + 6x + 5 = 0$

18. If the sum of the zeroes of the polynomial $p(x) = (k^2 - 14)x^2 - 2x - 12$ is 1, then find the value of k.

Sol. Given that, $p(x) = (k^2 - 14)x^2 - 2x - 12$

$$\text{Sum of the zeroes} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = 1$$

$$\Rightarrow \frac{-(-2)}{k^2-14} = 1 \Rightarrow \frac{2}{k^2-14} = 1$$

$\Rightarrow 2 = k^2 - 14$

$\Rightarrow k^2 = 16$

$\Rightarrow k = \pm\sqrt{16} \quad \Rightarrow \quad k = \pm 4$

SHORT ANSWER (SA) TYPE I QUESTIONS

19. Divide the polynomial $p(x)$ by the polynomial $q(x)$ and find the quotient and remainder. [NCERT]

$p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Sol. $p(x) = x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r|l}
 & -x^2 - 2 \\ \hline
-x^2 + 2 & x^4 + 0x^2 - 5x + 6 \\
 & \underset{-}{x^4} \underset{+}{-} 2x^2 \\ \cline{2-2}
 & \quad 2x^2 - 5x + 6 \\
 & \quad \underset{-}{2x^2} \qquad \underset{+}{-}4 \\ \cline{2-2}
 & \qquad -5x + 10 \\ \cline{2-2}
\end{array}$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

20. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial. [NCERT]

$t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Sol. $t^2 - 3 = t^2 + 0t - 3$

$$\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\ \hline
t^2 + 0t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & \underset{-}{2t^4} \underset{-}{+ 0t^3} \underset{+}{-} 6t^2 \\ \cline{2-2}
 & \qquad 3t^3 + 4t^2 - 9t - 12 \\
 & \qquad \underset{-}{3t^3} \underset{-}{+ 0t^2} \underset{+}{-} 9t \\ \cline{2-2}
 & \qquad\qquad 4t^2 + 0t - 12 \\
 & \qquad\qquad \underset{-}{4t^2} \underset{-}{+ 0t} \underset{+}{-} 12 \\ \cline{2-2}
 & \qquad\qquad\qquad 0 \\ \cline{2-2}
\end{array}$$

Since the remainder is 0,

therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

21. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial. [NCERT]

$x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Sol.

$$\begin{array}{r|l}
 & x^2 - 1 \\ \hline
x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & \underset{-}{x^5} \underset{+}{-} 3x^3 \underset{-}{+ x^2} \\ \cline{2-2}
 & \qquad -x^3 \qquad + 3x + 1 \\
 & \qquad \underset{+}{-x^3} \qquad \underset{-}{+ 3x} \underset{+}{- 1} \\ \cline{2-2}
 & \qquad\qquad\qquad\quad 2 \\ \cline{2-2}
\end{array}$$

Since, the remainder is $2 \neq 0$, therefore, $x^2 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

22. If m and n are the zeroes of the polynomial $ax^2 - 5x + c = 0$, find the values of a and c when $m + n = mn = 10$.

Sol. Quadratic polynomial is $ax^2 - 5x + c = 0$.

Sum of the zeroes, $m + n = 10$

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = 10$$

$$\Rightarrow \frac{-(-5)}{a} = 10 \Rightarrow \frac{5}{a} = 10 \Rightarrow a = \frac{5}{10} \Rightarrow a = \frac{1}{2}$$

Product of the zeroes, $mn = 10$

$$\frac{\text{Constant term}}{\text{Coefficient of } x^2} = 10$$

$$\Rightarrow \frac{c}{a} = 10 \Rightarrow c = 10a \Rightarrow c = 10 \times \frac{1}{2} \Rightarrow c = 5$$

SHORT ANSWER (SA) TYPE II QUESTIONS

23. Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients. [NCERT]

Sol. $4s^2 - 4s + 1 = (2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, *i.e.*, $s = \frac{1}{2}$.

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1$

$$= \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

Product of zeroes

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

24. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients. [NCERT]

Sol. $6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, *i.e.*, $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes

$$= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes

$$= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

25. Find the zeroes of the quadratic polynomial $t^2 - 15$ and verify the relationship between the zeroes and the coefficients. [NCERT]

Sol. $t^2 - 15$

$= t^2 - (\sqrt{15})^2 = (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, *i.e.*, when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0$

$$= \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

Product of zeroes $= (\sqrt{15})(-\sqrt{15})$

$$= -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } t^2}$$

26. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$. [NCERT]

Sol. $p(x) = x^3 - 3x^2 + x + 2$ (Dividend)

$g(x) = ?$ (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

$g(x)$ is the quotient when we divide

$(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$.

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x^2 \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

$\therefore g(x) = (x^2 - x + 1)$

27. If α and β are the zeroes of the polynomial $x^2 - 2x - 1$, then form a quadratic polynomial whose zeroes are $2\alpha - 1$ and $2\beta - 1$.

Sol. The given polynomial is $x^2 - 2x - 1$.

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-2)}{1} = 2$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-1}{1} = -1$$

For the given zeroes, $(2\alpha - 1)$ and $(2\beta - 1)$

Sum of the zeroes = $(2\alpha - 1) + (2\beta - 1)$

$= 2\alpha - 1 + 2\beta - 1$

$= 2\alpha + 2\beta - 2$

$= 2(\alpha + \beta) - 2$

$= 2 \times 2 - 2 = 4 - 2 = 2$

Product of the zeroes = $(2\alpha - 1)(2\beta - 1)$

$= 4\alpha\beta - 2\beta - 2\alpha + 1$

$= 4\alpha\beta - 2(\alpha + \beta) + 1$

$= 4(-1) - 2(2) + 1$

$= -4 - 4 + 1 = -7$

28. Find all the zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zeroes is 1.

Sol. If one of the zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$ is 1, then $(x - 1)$ is a factor of the given polynomial.

$$\begin{array}{r|l} & 3x^2 + 13x + 4 \\ \hline x - 1 & 3x^3 + 10x^2 - 9x - 4 \\ & 3x^3 - 3x^2 \\ & - \quad + \\ \hline & 13x^2 - 9x - 4 \\ & 13x^2 - 13x \\ & - \quad + \\ \hline & 4x - 4 \\ & 4x - 4 \\ & - \quad + \\ \hline & 0 \end{array}$$

Now, $(3x^3 + 10x^2 - 9x - 4)$

$= (x - 1)(3x^2 + 13x + 4)$

$= (x - 1)(3x^2 + 12x + x + 4)$

$= (x - 1)\{3x(x + 4) + 1(x + 4)\}$

$= (x - 1)\{(x + 4)(3x + 1)\}$

$= (x - 1)(x + 4)(3x + 1)$

If $x + 4 = 0$, then $x = -4$.

If $3x + 1 = 0$, then $x = \frac{-1}{3}$.

Therefore, all zeroes of the given polynomial are 1, –4, and $\frac{-1}{3}$.

29. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, find the value of $(\alpha^4\beta^2 + \alpha^2\beta^4)$.

Sol. We have, $f(x) = x^2 - 4x + 3$

Now, $\alpha + \beta$

$$= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a} = \frac{3}{1} = 3$$

$\alpha^4\beta^2 + \alpha^2\beta^4 = \alpha^2\beta^2(\alpha^2 + \beta^2)$

$= \alpha^2\beta^2\ (\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta)$

$= \alpha^2\beta^2\{(\alpha + \beta)^2 - 2\alpha\beta\} = (\alpha\beta)^2\ \{(\alpha + \beta)^2 - 2\alpha\beta\}$

$= (3)^2\{(4)^2 - 2 \times 3\} = 9\{16 - 6\} = 9 \times 10 = 90$

30. Find the value of *k* such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product.

Sol. We have, the given polynomial $x^2 - (k + 6)x + 2(2k - 1)$.

According to the question,

Sum of its zeroes = Half of product of zeroes

$$\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{1}{2}{}' \ \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \frac{-\{-(k+6)\}}{1} = \frac{1}{2} \times \frac{2(2k-1)}{1}$$

$$\Rightarrow \frac{k+6}{1} = \frac{2k-1}{1}$$

$\Rightarrow$ $k + 6 = 2k - 1$

$\Rightarrow$ $2k - k = 6 + 1$

$\Rightarrow$ $k = 7$

LONG ANSWER (LA) TYPES QUESTIONS

31. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes. [NCERT]

Sol. Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$= x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial.

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

$(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

i.e., $x = 7$ or -5

Hence, 7 and –5 are also zeroes of this polynomial.

32. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. [NCERT]

Sol: $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$$

is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$
\begin{array}{r l}
 & 3x^2 + 6x + 3 \\
\cline{2-2}
x^2 + 0x - \frac{5}{3} \,\Big) & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad\; - \qquad + \\
\cline{2-2}
 & \qquad 6x^3 + 3x^2 - 10x - 5 \\
 & \qquad 6x^3 + 0x^2 - 10x \\
 & \qquad - \quad\; - \qquad + \\
\cline{2-2}
 & \qquad\quad 3x^2 + 0x - 5 \\
 & \qquad\quad 3x^2 + 0x - 5 \\
 & \qquad\quad - \quad\; - \qquad + \\
\cline{2-2}
 & \qquad\qquad 0 \\
\cline{2-2}
\end{array}
$$

$3x^4 + 6x^3 - 2x^2 - 10x - 5$

$= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

We factorise $x^2 + 2x + 1 = (x + 1)^2$

Therefore, its zero is given by $x + 1 = 0$

$\Rightarrow x = -1$

As it has the term $(x + 1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, –1 and –1.